Tutorial: Random Variables

1. Classify each random variable as either discrete or continuous
   a. The number of boys in a randomly selected three-child family.
   b. The temperature of a cup of coffee served at a restaurant.
   c. The number of no-shows for every 100 reservations made with a commercial airline.
   d. The number of vehicles owned by a randomly selected household.
   e. The average amount spent on electricity each July by a randomly selected household in a certain state.
   f. Classify each random variable as either discrete or continuous
   g. The number of patrons arriving at a restaurant between 5:00p.m. and 6:00p.m
   h. The number of new cases of influenza in a particular county in a coming month.
   i. The air pressure of a tire on an automobile.
   j. The amount of rain recorded at an airport one day.
   k. The number of students who actually register for classes at a university next semester.

2. Identify the set of possible values for each random variable. (Make a reasonable estimate based on experience, where necessary.)
   a. The number of heads in two tosses of a coin.
   b. The average weight of newborn babies born in a particular county one month.
   c. The amount of liquid in a 12-ounce can of soft drink.
   d. The number of games in the next World Series (best of up to seven games).
   e. The number of coins that match when three coins are tossed at once.
   f. The number of hearts in a five-card hand drawn from a deck of 52 cards that contains 13 hearts in all.
   g. The number of pitches made by a starting pitcher in a major league baseball game.

h. The number of breakdowns of city buses in a large city in one week.

i. The distance a rental car rented on a daily rate is driven each day.

j. The amount of rainfall at an airport next month.

3. A discrete random variable X has the following probability distribution:

x	77	78	79	80	81
$P(x)$	0.15	0.15	0.20	0.40	0.10

Compute each of the following quantities.

a. P(80)

b. P(X>80).

c. P(X≤80).

d. The mean μ of X.

e. The variance σ^2 of X.

f. The standard deviation σ of X.

4. Tom works in an automotive tire factory. The number X of sound but blemished tires that he produces on a random day has the probability distribution

x	2	3	4	5
$P(x)$	0.48	0.36	0.12	0.04

a. Find the probability that Tom will produce more than three blemished tires tomorrow.

b. Find the probability that Tom will produce at most two blemished tires tomorrow.

c. Compute the mean and standard deviation of X. Interpret the mean in the context of the problem.

5. The number X of days in the summer months that a construction crew cannot work because of the weather has the probability distribution as follows

x	6	7	8	9	10	11	12	13	14
$P(x)$	0.03	0.08	0.15	0.20	0.19	0.16	0.10	0.07	0.02

   a. Find the probability that no more than ten days will be lost next summer.

   b. Find the probability that from 8 to 12 days will be lost next summer.

   c. Find the probability that no days at all will be lost next summer.

   d. Compute the mean and standard deviation of X. Interpret the mean in the context of the problem.

6. Five thousand lottery tickets are sold for $1 each. One ticket will win $1,000, two tickets will win $500 each, and ten tickets will win $100 each. Let X denote the net gain from the purchase of a randomly selected ticket.

   a. Construct the probability distribution of X.

   b. Compute the expected value E(X) of X. Interpret its meaning.

   c. Compute the standard deviation σ of X.

7. The time, to the nearest whole minute, that a city bus takes to go from one end of its route to the other has the probability distribution shown.

x	42	43	44	45	46	47
$P(x)$	0.10	0.23	0.34	0.25	0.05	0.02

   a. Find the average time the bus takes to drive the length of its route.

   b. Find the standard deviation of the length of time the bus takes to drive the length of its route.

8. John enters a local branch bank at 4:30p.m. every payday, at which time there are always two tellers on duty. The number X of customers in the bank who are either at a teller window or are waiting in a single line for the next available teller has the following probability distribution.

x	0	1	2	3	4	5	6
$P(x)$	0.135	0.192	0.284	0.230	0.103	0.051	0.005

   a. What number of customers does John most often see in the bank the moment he enters?

   b. What number of customers waiting in line does John most often see the moment he enters?

   c. What is the average number of customers who are waiting in line the moment John enters?

9. An insurance company will sell a $90,000 one-year term life insurance policy to an individual in a particular risk group for a premium of $478. Find the expected value to the company of a single policy if a person in this risk group has a 99.62% chance of surviving one year.

10. The owner of a proposed outdoor theater must decide whether to include a cover that will allow shows to be performed in all weather conditions. Based on projected audience sizes and weather conditions, the probability distribution for the revenue X per night if the cover is not installed is

Weather	x	$P(x)$
Clear	$3,000	0.61
Threatening	$2,800	0.17
LightRain	$1,975	0.11
Show – cancelling rain	$0	0.11

The additional cost of the cover is $410,000. The owner will have it built if this cost can be recovered from the increased revenue the cover affords in the first ten 90-night seasons.

a. Compute the mean revenue per night if the cover is not installed.

b. Use the answer to (a) to compute the projected total revenue per 90-night season if the cover is not installed.

c. Compute the projected total revenue per season when the cover is in place. To do so, assume that if the cover were in place the revenue each night of the season would be the same as the revenue on a clear night.

d. Using the answers to (b) and (c), decide whether or not the additional cost of the installation of the cover will be recovered from the increased revenue over the first ten years. Will the owner have the cover installed?

END OF TUTORIAL QUESTIONS