Tutorial: Continuous Random Variable

Watch this YouTube video to get a better understanding of the topic before attempting to the questions.

Continuous Random Variables: Probability Density Functions

https://www.youtube.com/watch?v=9KVR1hJ8SxI&t=772s

Question 1

A continuous random variable X has p.d.f

$f(x) = 5x^4, 0 \leq x \leq 1$

Find a_1 and a_2 such that

(i) $P[X \leq a_1] = P[X > a_1]$
(ii) (ii) $P[X > a_2] = 0.05$

Question 2

Suppose the life in hours of a radio tube has the following p.d.f

$$f(x) = \begin{cases} \frac{100}{x^2}, when\, x \geq 100 \\ 0, \quad when\, x < 100 \end{cases}$$

Find the distribution function.

Question 3

The time, in seconds, it takes to re-heat a cup of coffee can be modelled by the continuous random variable X, with probability density function.

$$f(x) = \begin{cases} \frac{3}{8}x^2, & 0 \le x \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate the mean amount of time it takes to re-heat a cup of coffee.

Question 4

The amount of bread (in hundreds of pounds) x that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon, with a probability function specified by the probability density function f (x) is given by

$$f(x) = \begin{cases} Ax, \text{ for } 0 \le x < 10 \\ A(20-x), \text{ for } 10 \le x < 20 \\ 0, \quad \text{otherwise} \end{cases}$$

a. Find the value of A.
b. What is the probability that the number of pounds of bread that will be sold tomorrow is
   i. More than 10 pounds,
   ii. Less than 10 pounds, and
   iii. Between 5 and 15 pounds?